Review of Essential Mathematics for Political and Social Research, by Jeff Gill

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Introduction

Jeff Gill's new Essential Mathematics for Political and Social Research (EMPSR) is one of few texts developed for political scientists, by a political scientist, that motivates the range of topics taught during a typical first graduate course in quantitative methods. Different programs involve different first courses – semester courses may focus on probability and mathematical statistics, data analysis and regression, or research methods broadly construed; one- or two-week pre-semester "math camps" also abound. EMPSR speaks to all of these settings, but focuses on short prefresher courses. After some general comments about usage in various settings, I trace EMPSR's contents more systematically, and compare EMPSR to other popular introductory materials.

EMPSR could serve as the primary or secondary text in many introductory settings. The range of topics suffices to fill a semester-long course on prerequisite mathematics, especially in departments where new graduate students may arrive with little quantitative background. Departments that begin graduate training with data analysis and regression could view the text as a prerequisite to program entry, and encourage summer study by their incoming classes. The wealth of applied examples could inform discussion in courses on general social science research methods, either at the graduate or undergraduate level. Given their breadth, however, such courses may not invest the time required for mastery of most mathematical skills EMPSR addresses. Math camp courses should seriously consider using EMPSR as their primary text, and Gill offers several templates for doing so. My experience with EMPSR is largely as an instructor of one such course; thus, although EMPSR can contribute in several settings, a math camp perspective dominates this review.

The students' diverse set of backgrounds, aptitudes, and interests creates much of the difficulty in teaching a successful first methods course in political science. In many programs, future philosophers sit next to future statisticians. This diversity particularly complicates a math camp textbook's two-fold charge: to enable all students to feel reasonably well-prepared on the first day of a term-time course, and to remain relevant beyond the first week of everyone's graduate career. EMPSR achieves both, by starting with extremely elementary material, incorporating over 100 examples (most drawn from actual, published social research), and still touching on topics that may not resurface until the third or fourth graduate course in statistical methods.

The numerous examples provide instructors with a ready-made answer to a common question of new graduate students: "Why am I starting political science grad school with pure math?" Namely, "To learn useful tools for answering interesting substantive questions in politics." Although the title suggests that EMPSR might be purely a primer in basic mathematics, the text highlights social science applications. Prominent examples follow in the next section.

New graduate students and those anticipating graduate study should find comfort in EMPSR's approachable style, its warnings of common confusions, and in the connections it draws between the mathematics and the substantive questions of interest. EMPSR's chapters begin with explicitly-stated objectives, helping to socialize new political scientists into the academic discipline. Reference tables, intuitive explanations, chapter lists of new terminology, and the relevance and volume of the topics all imply that EMPSR will be helpful in the early days, but also a well-worn text by the time students finish their degrees.

Concepts and Examples

EMPSR begins with the most elementary mathematical topics required for quantitative research: arithmetic, notation, and functions. However, by page 5 students have already encountered something that political scientists will recognize as being of significant value: Riker and Ordeshook (1968)s model for voter utility, R = PB - C. Although simple, this model is still widely discussed; in 2006 alone it has appeared in the APSR, AJPS, BJPS, and JOP. The second chapter covers analytic geometry and includes the most relevant topics from a high school trigonometry course. Political-scientific examples include parabolic presidential approval and elliptical voting preference models.

Chapters 3 and 4 straightforwardly introduce linear

algebra. Chapter 3 defines vectors, matrices, operations, and related properties. Central topics in Chapter 4 include the geometry of matrices, the determinant, eigenvalues, quadratic forms, and inverses. Example 4.6, a two-page exercise in estimating OLS regression parameters, provides an introduction to a ubiquitous application of matrix algebra to political science data.

Scalar and vector calculus fill Chapters 5 and 6, which include traditional definitions and applications of limits, derivatives, and integrals. Example 5.9 applies scalar calculus to another mainstay of political science, the Median Voter Theorem of Black (1958). EMPSR highlights oftenused skills such as extrema- and root-finding, multiple integration, and vector differentiation. Foundations such as the gradient and Hessian, Lagrange multipliers, and constrained optimization appear.

The pre-statistics section of EMPSR begins with probability theory in Chapter 7. True to his Bayesian roots, Gill opens the chapter with a discussion of subjective versus objective probability. This chapter includes set-theoretic definitions and properties, probability functions, conditional probability, Bayes' rule, Simpson's paradox, and independence. EMPSR also demonstrates odds, a topic that political scientists often encounter, but may have less prior exposure to than some other social researchers (like epidemiologists).

Chapter 8 covers random variables, and includes a 4page discussion of levels of measurement. EMPSR introduces familiar distributional families as models for social datagenerating processes: Bernoulli and binomial data, Poisson counts, and uniform, exponential, gamma, and normal Gaussian phenomena all appear. This chapter's wealth of applied modeling examples includes Supreme Court decisions, legislative bill passage, strategic alliance formation, incidence of war, income distributions, probit analysis of vote choice, and the presence of women in US state legislatures. The last of these features a quantile-quantile plot, thus giving a welcome introduction to model fit diagnostics.

Only after these examples does EMPSR cover measures of central tendency and spread. This ordering is consistent with texts like Rice (1995), but can lead to discussion of these measures that precedes their formal definition. EMPSR succeeds more than Rice in minimizing such discussion, but does not avoid it entirely as do treatments like Purves (1991). Other topics include summary statistics' breakdown points, correlation, expected value inequalities, and distributions' moments and central moments. A dierolling example illustrates expected value, as does an extended sequence of calculations derived from craps bets.

The last chapter is somewhat unexpected. Here, Gill introduces Markov chains, a topic that most political scientists might not encounter until they take a course in Bayesian modeling or data analysis. The chapter elucidates major chain concepts (periodicity, homogeneity, irreducibility, reversibility) and state characteristics (recurrent, absorbing, transient, closed). Gill demonstrates the utility of Markov chains as descriptive of social processes in their own right, but the underlying motivation may be to lay foundations for future Bayesian work.

Comparisons and Conclusions

There are many candidate materials for political science math camps. Instructor's notes, Simon and Blume (1994), Morgan (1997), and Hagle (1996) appear particularly common. EMPSR contrasts with Hagle and Morgan in two primary ways. First, EMPSR's scope is broader. The last third of EMPSR covers probability and statistics material omitted from the other two texts, for example. Second, the exercises and examples in EMPSR are significantly more applied than those of Hagle or Morgan.

One's preference for adopting EMPSR may hinge on whether one prefers the clear lines of fundamental skills repetition or the more thought-provoking and interpretive fuzziness of examples of political science research. To illustrate the difference, consider the problem sets on differentiation. Hagle's includes six consecutive questions instructing simply. "Find the derivatives of the following functions." Meanwhile, EMPSR sandwiches its one such question between exercises using published political research on suburban demographics and the siting of US county seats. Exercises in Hagle, Morgan, and Simon and Blume are also split into relatively small, homogeneous sections, while those of EMPSR are collected at chapters' ends. The former design encourages rote repetition, while the latter can obscure basic skills, but more accurately reflects the problems and choices students of methodology will face.

In my view, occasional over-complexity is the weakness of EMPSR. While the variety of examples is generally a strength of EMPSR, sometimes there is too much of a good thing. For example, in demonstrating inner products and cross products, Examples 3.6 and 3.8 use the same definitions for 1×3 vectors u and v, but Example 3.7 uses different ones. Using consistent definitions would simplify the matter and allow readers to focus on understanding the algebra. For a math camp, EMPSR's problem sets are long and include some potentially intimidating problems. Selecting exercises to assign will require judicious consideration of one's audience. Also, at this time, an answer key is still in development. Until its release, instructors may have to supply their own solutions.

On the whole, EMPSR succeeds. Its range and depth of topics form appropriate standards for incoming and continuing political science graduate students. Its constant attention to published research introduces budding professionals to exactly how and why learning mathematics is an important first step.

References

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Section Activities

A note from our Section President

I would like to offer a hardy thank you to Adam Berinsky, Michael Herron, and Jeff Lewis for their hard work and dedication in producing *The Political Methodologist* for the past three years! It is an invaluable communication tool for the section. Please join me in welcoming the new *TPM* editors, Paul Kellstedt, David Peterson, and Guy Whitten. We look forward to continued success of *TPM* under the guidance of the new Texas A&M editors.

Nominations for the second annual John T. Williams Dissertation Prize are being solicited. The prize is given in recognition of John T. Williams' contribution to graduate training and is for the best dissertation proposal in the area of political methodology. Proposals using quantitative or qualitative methods are welcomed and should follow the National Science Foundation length and format guidelines. Members of the committee are John Aldrich (chair), Tse-Min Lin, and Michael Colaresi. Materials should be sent to the John Aldrich at aldrich@duke.edu.

The 24th Annual Summer Meeting of the Society for Political Methodology will be held at Pennsylvania State University, July 19-21. The hosts, Suzanna DeBoef and Burt Monroe, have information about the conference available on the conference website at: http://polmeth.psu. edu/. The past success and popularity of the meetings have led the Society's membership to support the recommendation of the Long Range Planning Committee by implementing an alternative model for the meeting to accommodate increased demand. In an effort to extend participation, the meeting size is growing substantially. With increased size, however, come some inevitable changes. The basic program format and the venerated graduate student poster session will remain. The host institution will be providing breakfast and lunch for the participants throughout the conference and will host a dinner and a reception in 2007. All other expenses (notably, hotel accommodations and remaining dinners) will be covered by attendees. Registration applications are available at: http://polmeth.wustl.edu/ methods2007/register/. We thank the program committee, Rebecca Morton (chair), Suzanna DeBoef, Burt Monroe, Kevin Quinn, and Jake Bowers for their hard work and dedication in bringing together the meeting. The National Science Foundation, in conjunction with Penn State University, will continue to support 35 graduate students through a competitive process. We thank the Graduate Student Selection Committee for their work as well. The committee includes Dan Wood (chair), Michele Claiborne, David Darmofal, and Kevin Clarke.

We now have over sixteen active committees. A full listing of all the committee members and a list of their charges is available on the Political Methodology website: http://polmeth.wustl.edu/society.php. We thank for Andrew Martin and Stephen Haptonstahl at Washington University for their work on the website. They provide ths excellent service for the section gratis. I want to highlight one new committee, the Undergraduate and Graduate Methodology Committee, which is chaired by Lonna Atkeson. Other committee members include Garrett Glasgow, Paul Gronke, Dean Lacy, and Alan Zuckerman. Agenda items include: 1) developing best practices for departments and students in order to be prepared for graduate school in political science; 2) increase the availability of methods syllabi; 3) sponsor panels at the APSA Teaching and Learning Conference on undergraduate and graduate methods; 4) brainstorm about what the section should be doing for it's members who are at schools where there is significant emphasis on undergraduates and teaching; 5) explore best practices for interdisciplinary methods training for graduate students. Please contact them if you want to get involved or have an idea to share with them.

Best wishes,

Jan Box-Steffensmeier The Ohio State University